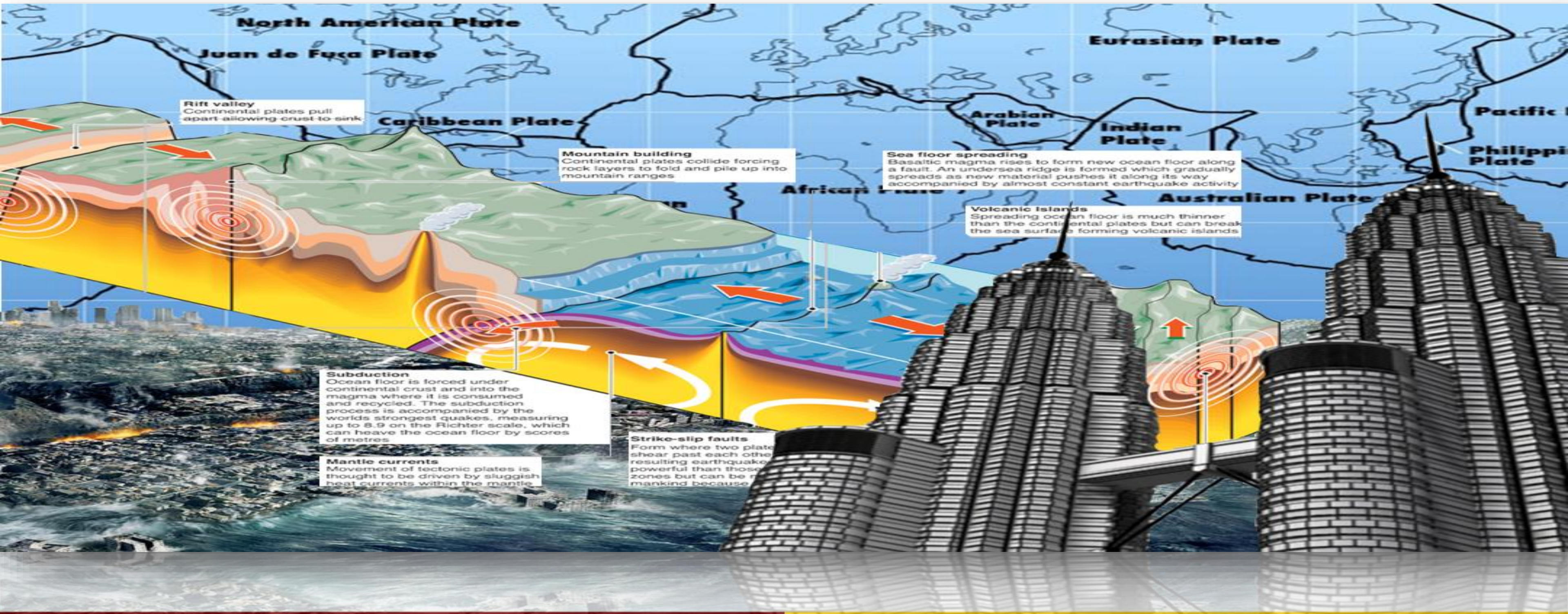
SEAA 3243

THEORY OF STRUCTURES
Slope Deflection Method
-Assoc. Prof. Ts. Dr. Sophia C. Alih-



Course Outlines

Introduction:

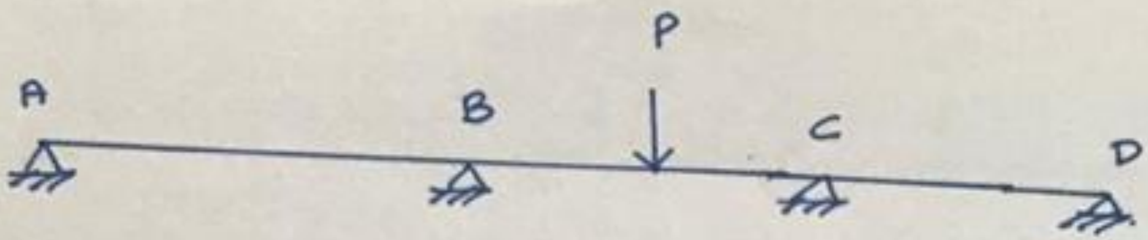
i. Type of structures – beams, frame, trusses
ii. Types of loads – point loads, uniform distributed load, moment
iii. Stability and determinacy

STATICALLY INDETERMINATE TRUSS, BEAMS AND FRAMES

1. Slope deflection method –> beams --- no settlement
   -- with settlement
   -> Frames -- no settlement
   -- with settlement

2. Moment distribution method –> beams -- no settlement
   -- with settlement
   -> Frames -- no settlement
   -- with settlement

TEST

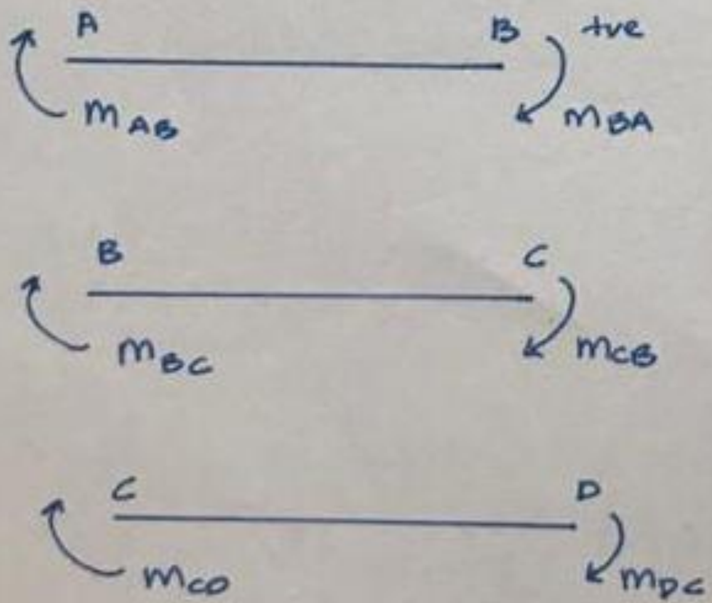


$r = 8$, $c+3 = 3$, $r > c+3 \Rightarrow$ indeterminate beam

Degree of indeterminacy $= 8 - 3 = \underline{5}$

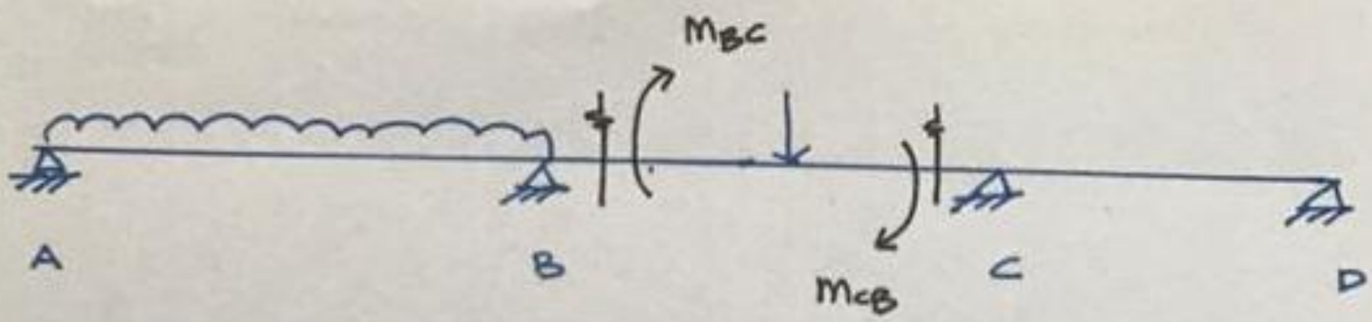
<u>End moments,</u>

- In general, there are 2 end moments acting on a member
- if we cut a member near the support, there'll be moment acting at each end,



⇒ At the begining, moment is assumed to be positive (↻)

⇒ Using slope deflection method, we want to calculate all these moments → to obtain the slope deflection equations



Once we get general equation for member BC, we can use for others.

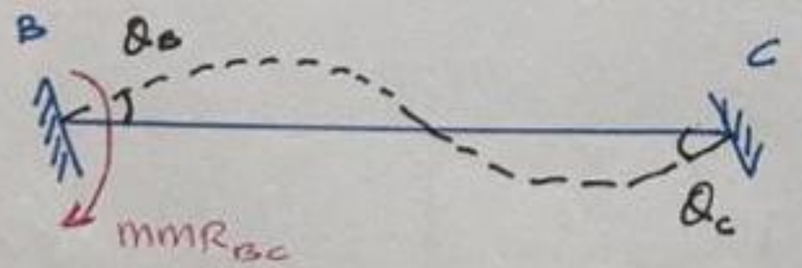
There are 3 types of moment acting at each end;

i) moment due to member Rotation, MMR

ii) moment due to support settlement, MSS

iii) Fixed end moment, FEM

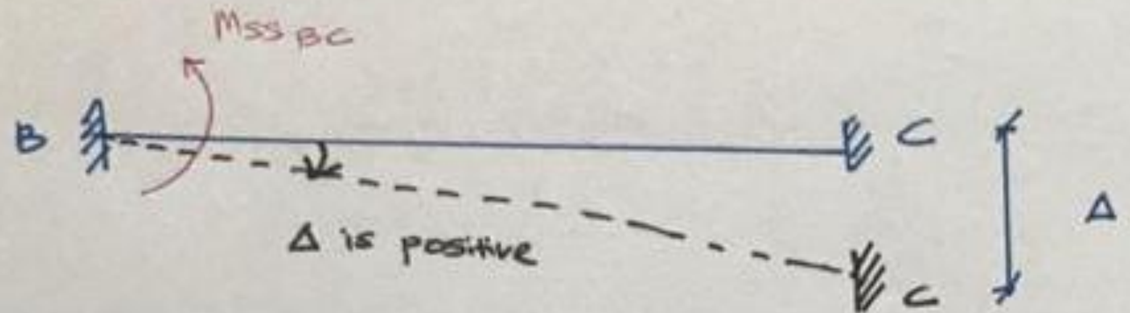
i) MMR : moment due to member rotation;



(direction: to push member back to original position)

$$\boxed{MMR_{BC} = \frac{2EI}{L}\left(2\theta_B + \theta_C\right)}$$

ii) **Mss** : moment due to support settlement;

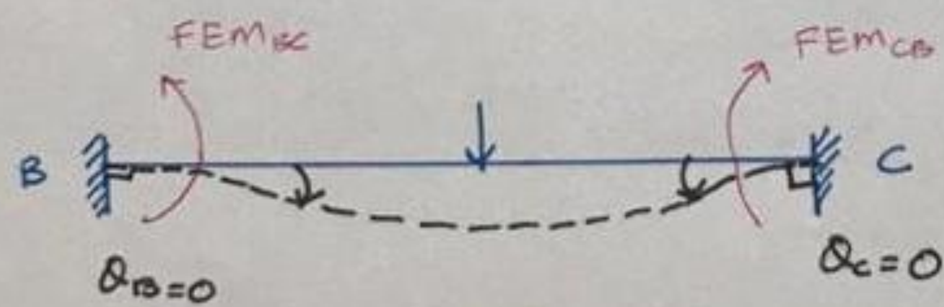


(direction: opposite to settlement)

$$\boxed{Mss_{BC} = \frac{-6EI\,\Delta}{L^2}}$$

(Δ is positive is member BC rotate clockwise) i(i) FEM : Fixed end moment

- Depends on the load on member



(no deflection at support coz it's fixed support; no angle of rotation)

FEM_{BC} ⇒ opposite the direction of member rotation.

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④

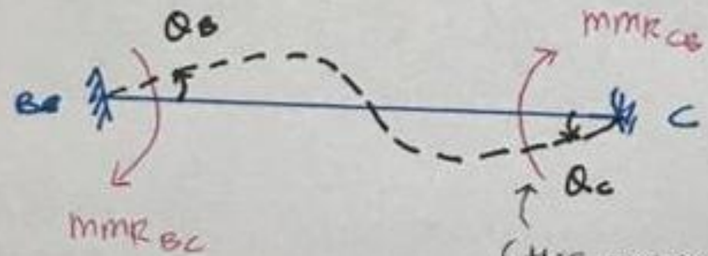
$$M_{BC} = MMR_{BC} + M_{SS\,BC} + FEM_{BC}$$

$$= \frac{2EI}{L}(2\theta_B + \theta_C) + \left[\frac{-6EI\Delta}{L^2}\right] + FEM_{BC}$$

$$\boxed{M_{BC} = \frac{2EI}{L}\left(2\theta_B + \theta_C - \frac{3\Delta}{L}\right) + FEM_{BC}}$$

$$M_{CB} = MMR_{CB} + MSS_{CB} + FEM_{CB}$$

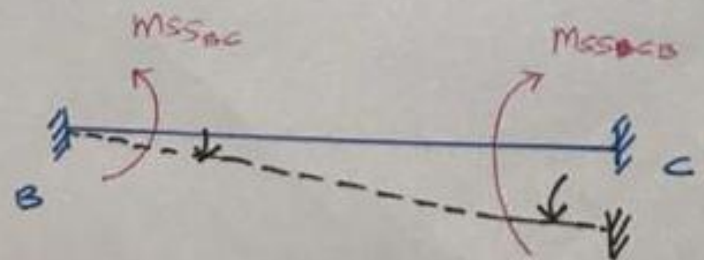
$\underline{MMR_{CB}}$



(this moment tried to push the member straight again)

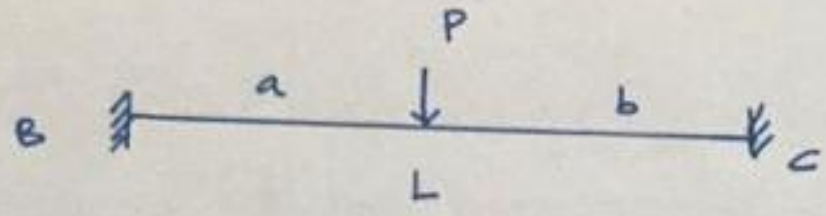
$$\boxed{MMR_{CB} = \frac{2EI}{L}\left(2Q_C + Q_B\right)}$$

$\underline{MSS_{CB}}$



$$\boxed{MSS_{CB} = -\frac{6EI\Delta}{L^2}}$$

$\underline{FEM_{CB}}$ ⇒ (Use Formula)



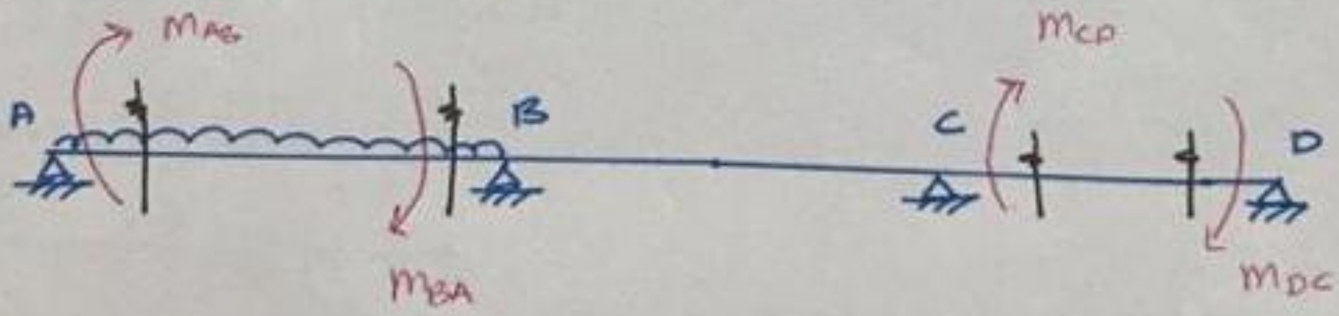
$$t = \frac{Pa^2 b}{L}$$

$$\therefore \; M_{CB} = MMR_{CB} + MSS_{CB} + FEM_{CB}$$

$$= \frac{2EI}{L}(2\theta_C + \theta_B) - \frac{6EI\Delta}{L^2} + FEM_{CB}$$

$$\boxed{M_{CB} = \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\Delta}{L}\right) + FEM_{CB}}$$

For member AB and CD,



$$M_{AB} = \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right) + FEM_{AB}$$

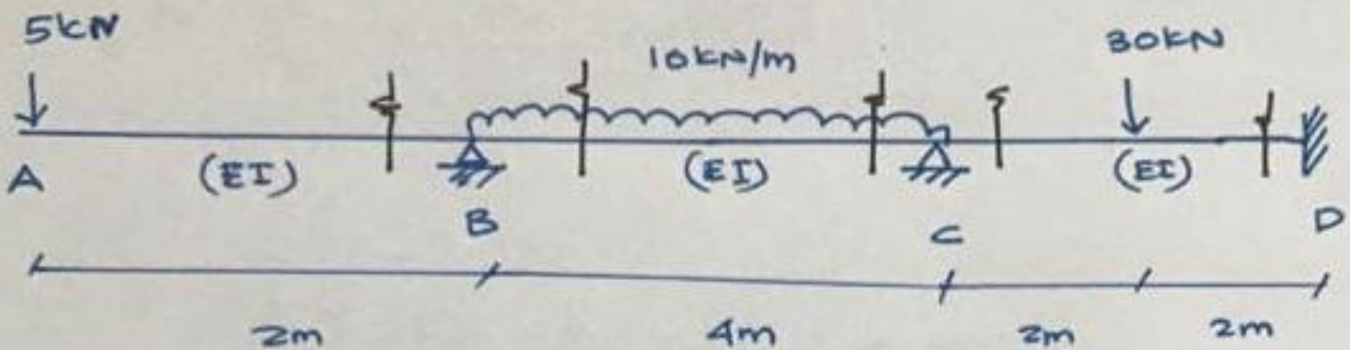
$$M_{BA} = \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\Delta}{L}\right) + FEM_{BA}$$

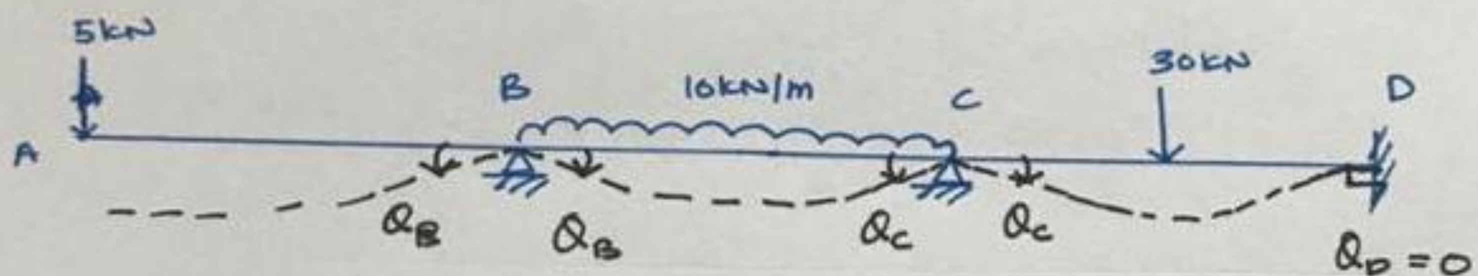
$$M_{CD} = \frac{2EI}{L}\left(2\theta_C + \theta_D - \frac{3\Delta}{L}\right) + FEM_{CD}$$

$$M_{DC} = \frac{2EI}{L}\left(2\theta_D + \theta_C - \frac{3\Delta}{L}\right) + FEM_{DC}$$

Exp :- slope deflection method

(Beam : no settlement)

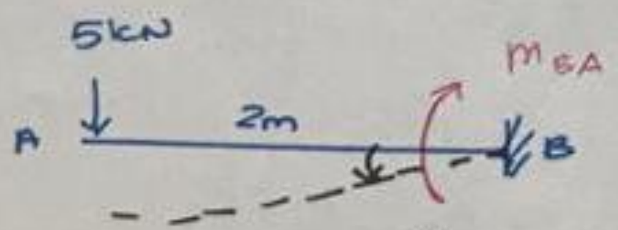




(near to fixed end
↳ no rotation)

(need to cut each member and solve one by one)

For member AB (cantilever)

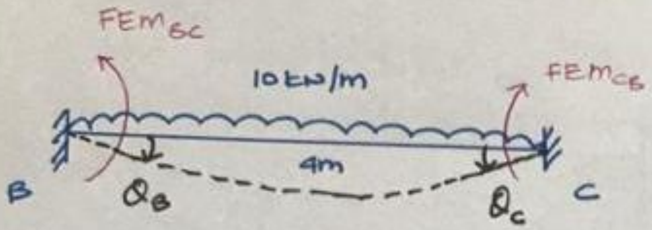


(opposite direction with beam)

$M_{BA} = +(5 \times 2)$

$= \underline{+10 \text{ kNm}}$ ——— (1)

(positive coz the direction is clockwise) member BC



$$\boxed{FEm = \frac{wL^2}{12}}$$

$$M_{BC} = \frac{2EI}{L}\left(2\theta_B + \theta_C - \frac{3\Delta}{L}\right) + FEM_{BC}$$

0 (no settlement)

$$= \frac{2EI}{4}\left(2\theta_B + \theta_C - \cancel{\frac{3\Delta}{L}}\right) - \frac{10(4)^2}{12}$$

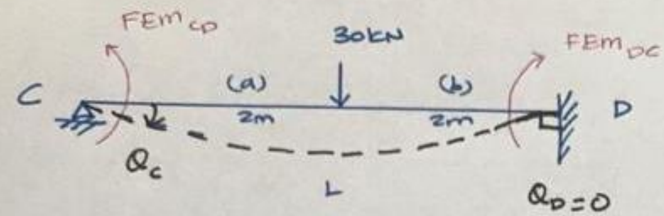
$$= EI\theta_B + \frac{EI\theta_C}{2} - 13.33 \quad \text{———} \quad (2)$$

$$M_{CB} = \frac{2EI}{L}\left(2\theta_C + \theta_B - \cancel{\frac{3\Delta}{L}}^{\,0}\right) + FEM_{CB}$$

$$= \frac{2EI}{4}\left(2\theta_C + \theta_B\right) + \frac{10(4)^2}{12}$$

$$= EI\theta_C + \frac{EI\theta_B}{2} + 13.33 \quad \text{———} \quad (3)$$

member CD



$$FEm_{CD} = \frac{-Pab^2}{L^2}$$

$$FEm_{DC} = \frac{+Pa^2b}{L^2}$$

$$m_{CD} = \frac{2EI}{L}\left(2Q_C + Q_D - \cancel{\frac{3\Delta}{L}}^{\,0\text{ (no settlement)}}\right) + FEm_{CD}$$

0 (near to fix support → reaction is zero)

$$= \frac{2EI}{4}\left(2Q_C + \cancel{Q_D}\right) + \left[-\frac{30\times2\times2^2}{4^2}\right]$$

$$= EIQ_C - 15 \quad \text{——— (4)}$$

$$m_{DC} = \frac{2EI}{L}\left(2Q_D + Q_C - \frac{3\Delta}{L}\right) + FEm_{DC}$$

$$= \frac{2EI}{4}\left(Q_C\right) + \left[\frac{30\times2^2\times2}{4^2}\right]$$

$$= \frac{EIQ_C}{2} + 15 \quad \text{——— (5)}$$

* need to calculate Q_B and Q_C first.

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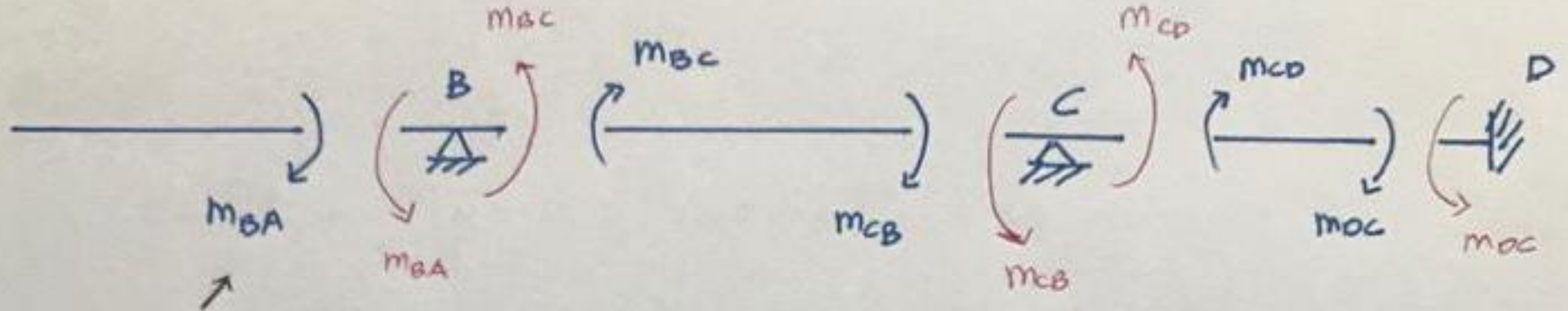
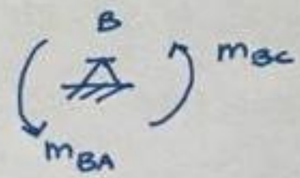
moment acting at support is always in equilibrium ;



(All have been calculated / derived)

(← direction is assumed to be positive)

Equilibrium of moment at B,



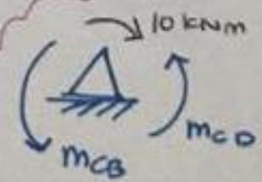
$$m_{BA} + m_{BC} = 0$$

$$10 + \left[EI\theta_B + \frac{EI\theta_C}{2} - 13.33 \right] = 0$$

$$EI\theta_B + \frac{EI\theta_C}{2} - 3.33 = 0$$

$$2EI\theta_B + EI\theta_C - 6.66 = 0 \quad \text{——— (6)}$$

Equilibrium of moment at C,



Exp:-
If there is external moment it needs to be included as well
$\Rightarrow m_{CB} + m_{CD} - 10 = 0$

$$m_{CB} + m_{CD} = 0$$

$$EI\theta_C + \frac{EI\theta_B}{2} + 13.33 + \left[EI\theta_C - 15 \right] = 0$$

$$2EI\theta_C + \frac{EI\theta_B}{2} - 1.67 = 0$$

$$EI\theta_B + 4EI\theta_C - 3.34 = 0 \quad \text{——— (7)}$$

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From 6,

$$EI\theta_C = 6.66 - 2EI\theta_B \text{ ——— } (8)$$

8 + 7,

$$EI\theta_B + 4(6.66 - 2EI\theta_B) - 3.34 = 0$$

$$\therefore \left.\begin{aligned} EI\theta_B &= 3.33 \\ EI\theta_C &= 0.003 \end{aligned}\right\} \text{ Subsitute these into Eq. } (2) \rightarrow (5),$$

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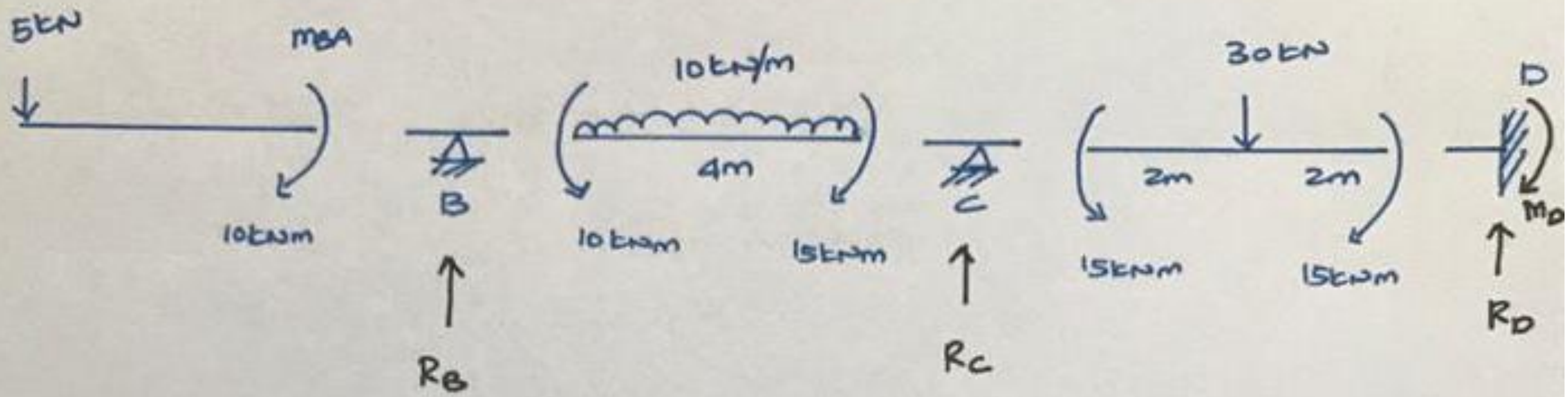
From ②,
$$M_{BC} = EI\theta_B + 0.5EI\theta_C - 13.33$$
$$= 3.33 + 0.5(0.003) - 13.33$$
$$= \underline{-10 \text{ kNm}}$$

From ③,
$$M_{CB} = EI\theta_C + 0.5EI\theta_B + 13.33$$
$$= 0.003 + 0.5(3.33) + 13.33$$
$$= \underline{+15 \text{ kNm}}$$

From ④,
$$M_{CD} = EI\theta_C - 15$$
$$= 0.003 - 15$$
$$= \underline{-15 \text{ kNm}}$$

From ⑤,
$$M_{DC} = \frac{EI\theta_C}{2} + 15$$
$$= (0.5 \times 0.003) + 15$$
$$= \underline{+15 \text{ kNm}}$$



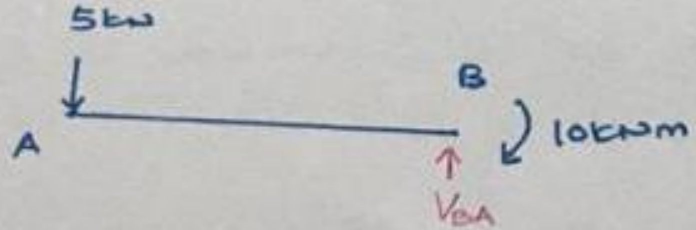
we need to calculate support reaction ; R_B, R_C, R_D, M_D

- At first all arrow is assumed to be positive.
- once we get the real value, then change the direction

→ to solve this, we need to calculate end support first.

To get Support reactions ;

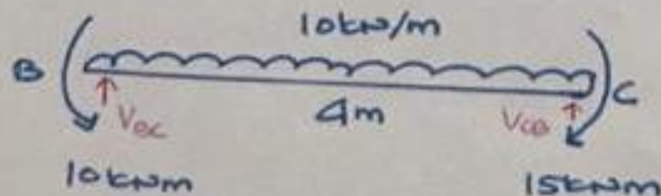
member AB ,



+ve $\uparrow$, $\Sigma F_y = 0$,

$-5 + V_{BA} = 0$, $V_{BA} = \underline{5\text{kN}}$ (must be balanced with another vertical load at the other end of the beam)

member BC



+ve $\circlearrowleft$ $\underline{\Sigma M_c = 0}$, $4V_{BC} - 10(4)(2) - 10 + 15 = 0$

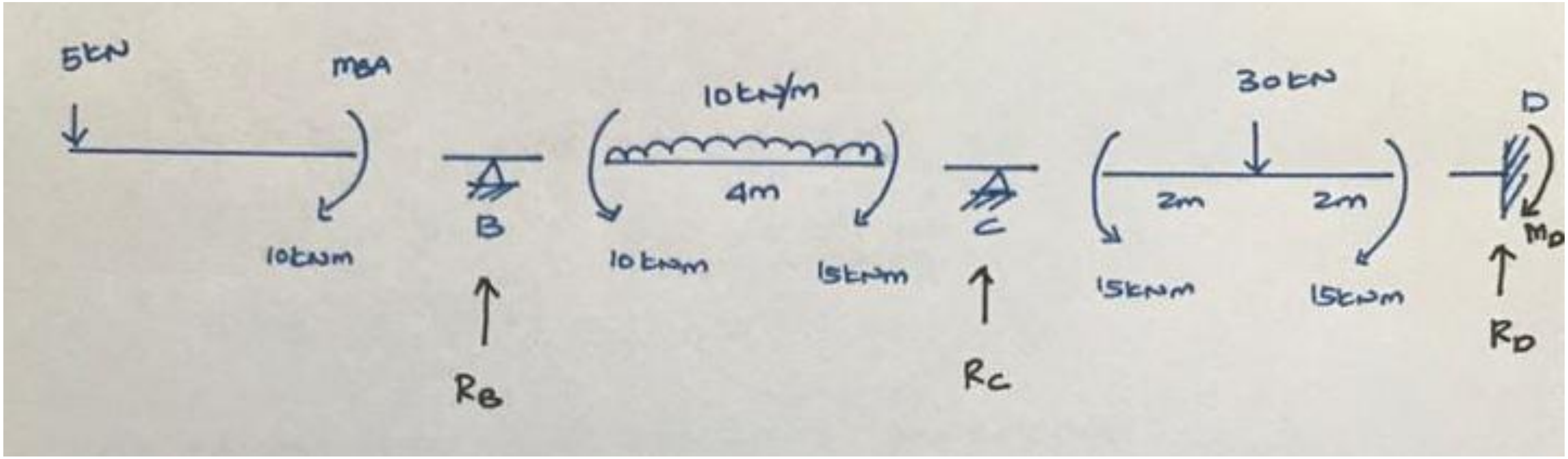
End moment at B

End moment at C

$V_{BC} = \underline{18.75}$ kN (must be balance at B support)

+ve $\uparrow$ $\underline{\Sigma F_y = 0}$, $18.75 - 10(4) + V_{CB} = 0$

$V_{CB} = \underline{21.25}$ kN



5kN
MBA
10kNm
B
RB
10kN/m
4m
10kNm
15kNm
C
RC
30kN
2m
2m
15kNm
15kNm
D
MD
RD

∴ Calculate Support Reaction at B,

+ve ↑ $\underline{\Sigma F_y = 0}$, $R_B - 5 - 18.75 = 0$

$$\underline{\underline{R_B = 23.75 \text{ kN}}}$$

<u>For member CD</u>, (all the same procedures)

$V_{CD} = 15\text{kN}$, $V_{DC} = 15\text{kN}$

To get R_C, $\Sigma F_y = 0$ at support C,

$R_C - 21.25 - 15 = 0$, $\underline{\underline{R_C = 36.25 \text{ kN}}}$

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$R_D = 15kN$ (to balance with the 15kN at end support)

⇒ To get moment at support D, at the end of member D, there should be another moment anti-clockwise (↺ 15kNm) to balance moment at the other support (member CD)

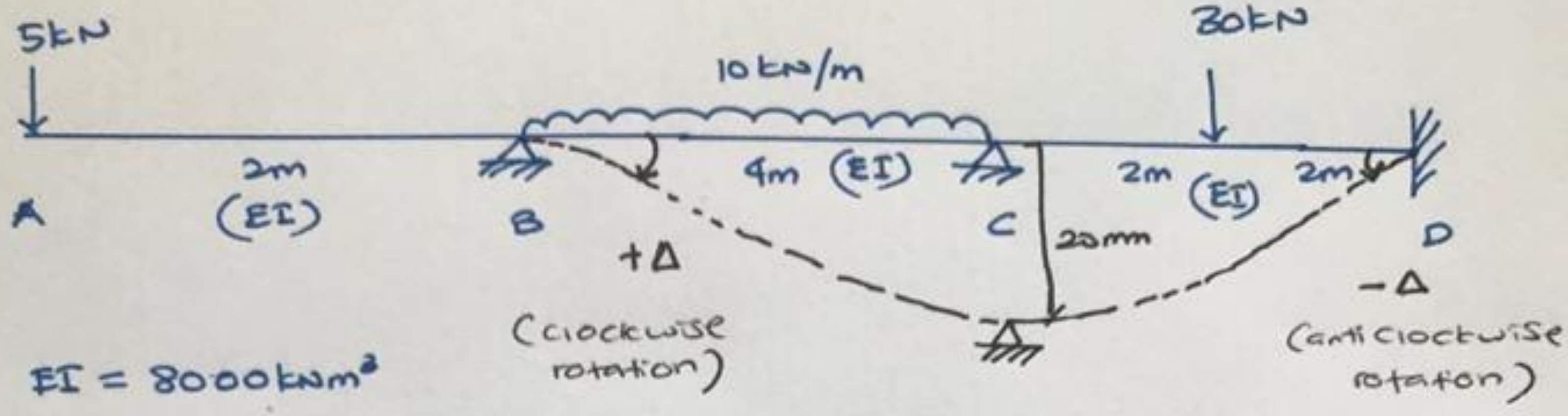
∴ For equilibrium at support D only ; $M_D = 15kNm$

* Once all support reaction have been calculated, shear force diagram, (SFD) and Bending moment diagram (BMD) can be prepared.

SFD & BMD :
Please refer to the video

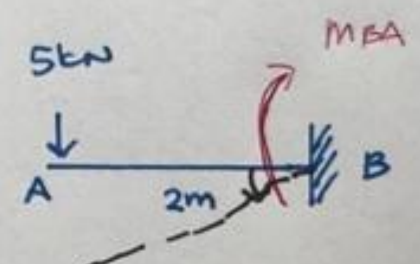
Exp: Slope deflection method – beam with settlement

Exp : Beam with settlement



$EI = 8000 kNm^2$

- There is support settlement at support C of 20mm
- Calculate the end moment for all members by using slope deflection method.



$M_{AB} = 0$

$M_{BA} = 5 \times 2m$

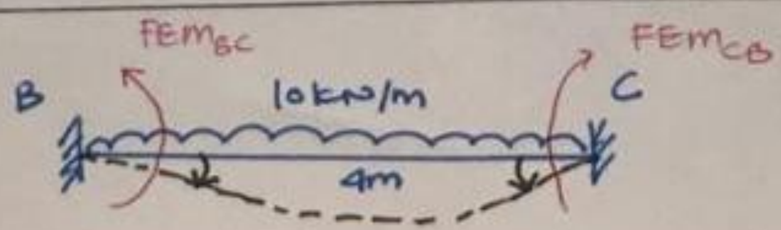
$= 10 kNm$ ——— (1)

$$M_{BC} = \frac{2EI}{L}\left(2\theta_B + \theta_C - \frac{3\Delta}{L}\right) + FEM_{BC}$$

$$= \frac{2EI}{4}\left(2\theta_B + \theta_C - \frac{3(+0.02)}{4}\right) + \frac{-10(4^2)}{12}$$

$$= \frac{2EI}{4}\left(2\theta_B + \theta_C - 0.015\right) - 13.33$$

$$= EI\theta_B + 0.5EI\theta_C - 0.0075EI - 13.33 \quad (2)$$



$$FEM_{BC} = \frac{-10L^2}{12}$$

$$M_{CB} = \frac{2EI}{4}\left(2\theta_C + \theta_B - \frac{3(0.02)}{4}\right) + FEM_{CB}, \quad FEM_{CB} = wL^2/12$$

$$= \frac{10\times4^2}{12}$$

$$= 13.33$$

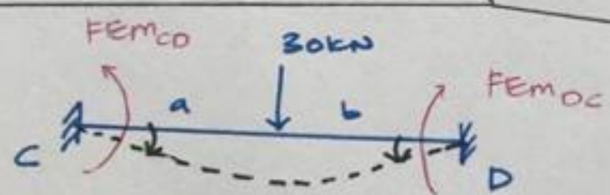
$$= EI\,\theta_C + 0.5EI\theta_B - 0.0075EI + 13.33 \quad \text{— (3)}$$

$$M_{CD} = \frac{2EI}{4}\left(2\theta_C + \theta_D - \frac{3\Delta}{L}\right) + FEM_{CD}$$

$$= EI\left(\theta_C + 0.5\theta_D - \frac{3(-0.02)}{4}\right) - 15$$

$$= EI\,\theta_C + 0.5EI\,\theta_D + 0.0075EI - 15 \quad \text{— (4)}$$

$\underline{FEM_{CD}}$



$$FEM_{CD} = -Pab^2/L^2$$

$$= -30\times2\times2^2/4^2$$

$$= -15\text{kNm}$$

$$FEM_{DC} = Pa^2b/L^2$$

$$= 30\times2^2\times2/4^2$$

$$= 15\text{kNm}$$

$$M_{DC} = \frac{2EI}{4}\left(2\theta_D + \theta_C - \frac{3\Delta}{L}\right) + FEM_{DC}$$

$$= \frac{EI}{2}\left(2\theta_D + \theta_C - \frac{3(-0.02)}{4}\right) + 15$$

$$= EI\theta_D + 0.5EI\theta_C + 0.0075EI + 15 \quad \text{—— (5)}$$

$\Rightarrow$ need to calculate θ_B, θ_C first ($\theta_D = 0$)

Similar steps:

1. Use equilibrium of moment at support to get θ_B and θ_C
2. Solve the equations to get Mbc, Mcb, Mcd, Mdc
3. Use equilibrium of forces at support to get support reactions Rb, Rc, Rd, and Md
4. Draw SFD and BMD

Assignment

Complete the calculation and Draw SFD and BMD for the example of beam with settlement

Due date: 29 March 2026– 5pm

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Q5. An indeterminate beam structure has a fixed support at A and roller supports at B and C as shown in Figure Q5. The beam is loaded as illustrated. The roller support at B settles 20 mm downward. Value of *EI* is equal to 6000 kNm^2. By using slope deflection method analyze the beam and answer the following questions:

(a) Determine the end moments for all members.

(10 marks)

(b) Determine all reaction forces.

(4 marks)

(c) Draw the shear force diagram for the beam.

(4 marks)

(d) Draw the bending moment diagram for the beam.

(4 marks)

(22 marks)

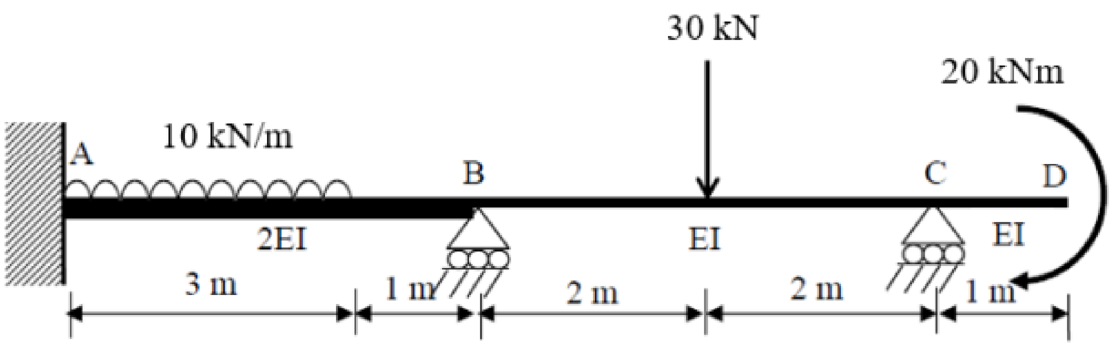
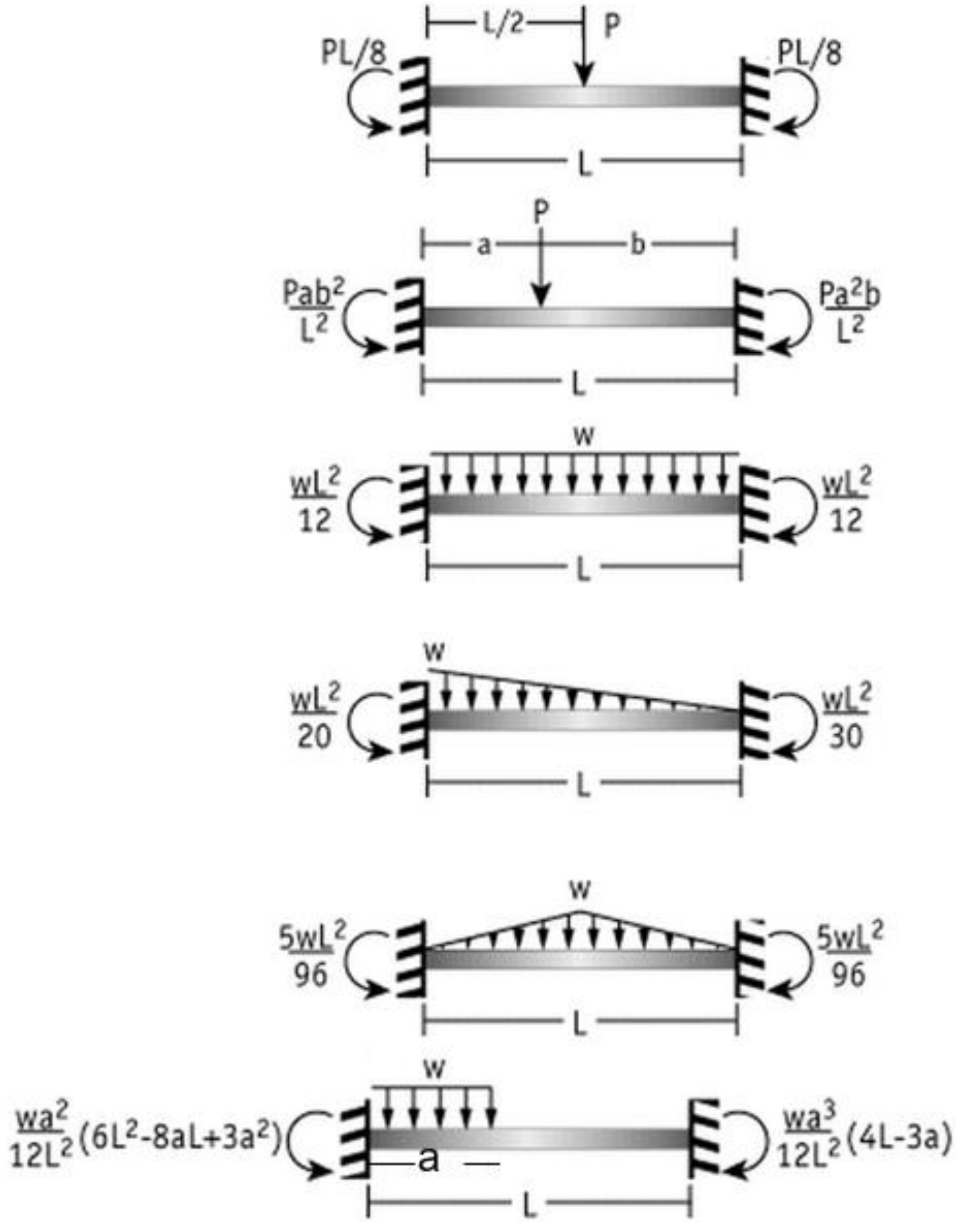
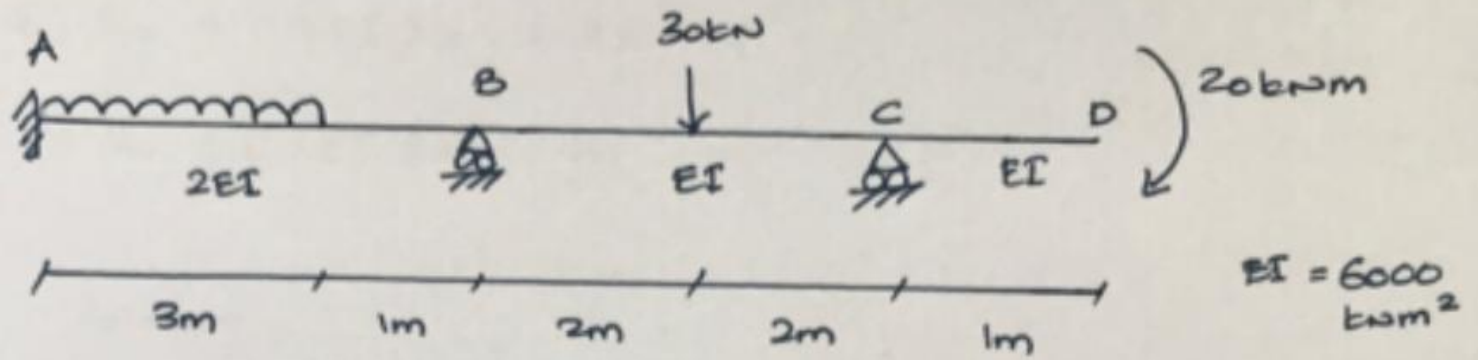


FIGURE Q5

Table of Fixed End Moment:





$EI = 6000\ kNm^2$

$\Delta_{eB} = 0.02m$

$$M_{AB} = \frac{2(2EI)}{4}\left(2\theta_A + \theta_B - \frac{3(0.02)}{4}\right) - \frac{10(3^2)}{12(4^2)}\left[6(4^2) - 8(3\times4) + 3(3^2)\right]$$

$$= EI\theta_B - 0.015EI - 12.66$$

$$= EI\theta_B - 0.015(6000) - 12.66$$

$$= EI\theta_B - 102.66 \quad \text{——— (1)} \qquad (2)$$

$$M_{BA} = \frac{2(2EI)}{4}\left(2\theta_B + \theta_A - \frac{3(0.02)}{4}\right) + \frac{10(3^3)}{12(4^2)}\left((4\times4) - (3\times3)\right)$$

$$= 2EI\theta_B - 0.015EI + 9.84$$

$$= 2EI\theta_B - 80.156 \text{ ——— } ②$$

2

$$M_{BC} = \frac{2EI}{4}\left[2\theta_B + \theta_C - \frac{3(-0.02)}{4}\right] - \frac{30(4)}{8}$$

$$= EI\theta_B + 0.5EI\theta_C + \frac{0.015EI}{2} - 15$$

2

$$= EI\theta_B + 0.5EI\theta_C + 30 \text{ ——— } ③$$

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$$M_{CB} = \frac{2EI}{4}\left[2\theta_C + \theta_B - \frac{3(-0.02)}{4}\right] + \frac{30(4)}{8}$$

$$= EI\theta_C + 0.5EI\theta_B + 45 + 15$$

$$= EI\theta_C + 0.5EI\theta_B + 60 \quad \text{——— (4)}$$

2

$$M_{CD} = -20\,kNm$$

$\Sigma M_{@B} = 0,$

$$M_{BA} + M_{BC} = 0,$$

$$2EI\theta_B - 80.156 + EI\theta_B + 0.5EI\theta_C + 30 = 0$$

$$3EI\theta_B + 0.5EI\theta_C - 50.16 = 0$$

$$6EI\theta_B + EI\theta_C - 100.31 = 0 \quad \text{——— (5)}$$

$\Sigma M_{C} = 0,$

$$M_{CB} + M_{CD} = 0$$

$$EI\theta_C + 0.5EI\theta_B + 60 - 20 = 0$$

$$EI\theta_C = -0.5EI\theta_B - 40 \text{ --- } (6)$$

$(6) \rightarrow (5),$

$$6EI\theta_B - 0.5EI\theta_B - 40 - 100.31 = 0$$

$$EI\theta_B = \frac{140.31}{5.5} = 25.51$$

$$\theta_B = \frac{25.51}{6000} = 0.004252 \text{ rad } (4.252 \times 10^{-3} \text{ rad})$$

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$$EI\,\theta_C = -0.5EI\,\theta_B - 40$$

$$= -0.5\,(25.51) - 40$$

$$\theta_C = \frac{-52.756}{6000}$$

$$= -0.008793 \text{ rad} \quad (8.793\times10^{-3} \text{ rad})$$

$\therefore$ End moments;

$$M_{AB} = EI\,(0.004252) - 102.66$$

$$= \underline{-77.14 \text{ kNm}}$$

$$M_{BA} = 2EI(0.004252) - 80.156$$

2

$$= \underline{-29.13 \text{ kNm}}$$

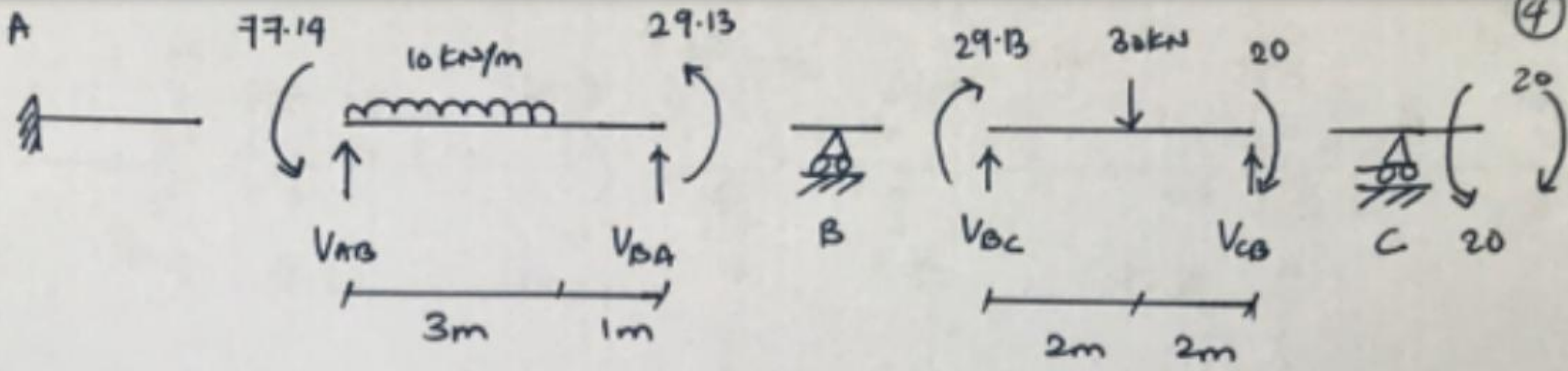
$$M_{BC} = EI(0.004252) + 0.5EI(-0.008793) + 30$$

$$= \underline{29.13 \text{ kNm}}$$

$$M_{CB} = EI(-0.008793) + 0.5EI(0.004252) + 60$$

$$= \underline{20 \text{ kNm}}$$

$$M_{CD} = \underline{-20 \text{ kNm}}.$$



member AB.

$\Sigma M_B = 0,$

$$V_{AB}(4) - 10(3)(2.5) - 77.14 - 29.13 = 0,$$

$$V_{AB} = 45.34 \text{ kN}$$

$\Sigma F_y = 0,$

$$V_{BA} + 45.34 - 10(3) = 0,$$

$$V_{BA} = -15.34 \text{ kN}$$

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Member BC,

2

$\Sigma M_{BC} = 0,$

$V_{BC}(4) - 30(2) + 29.13 + 20 = 0,$

$V_{BC} = 2.72 \text{ kN}$

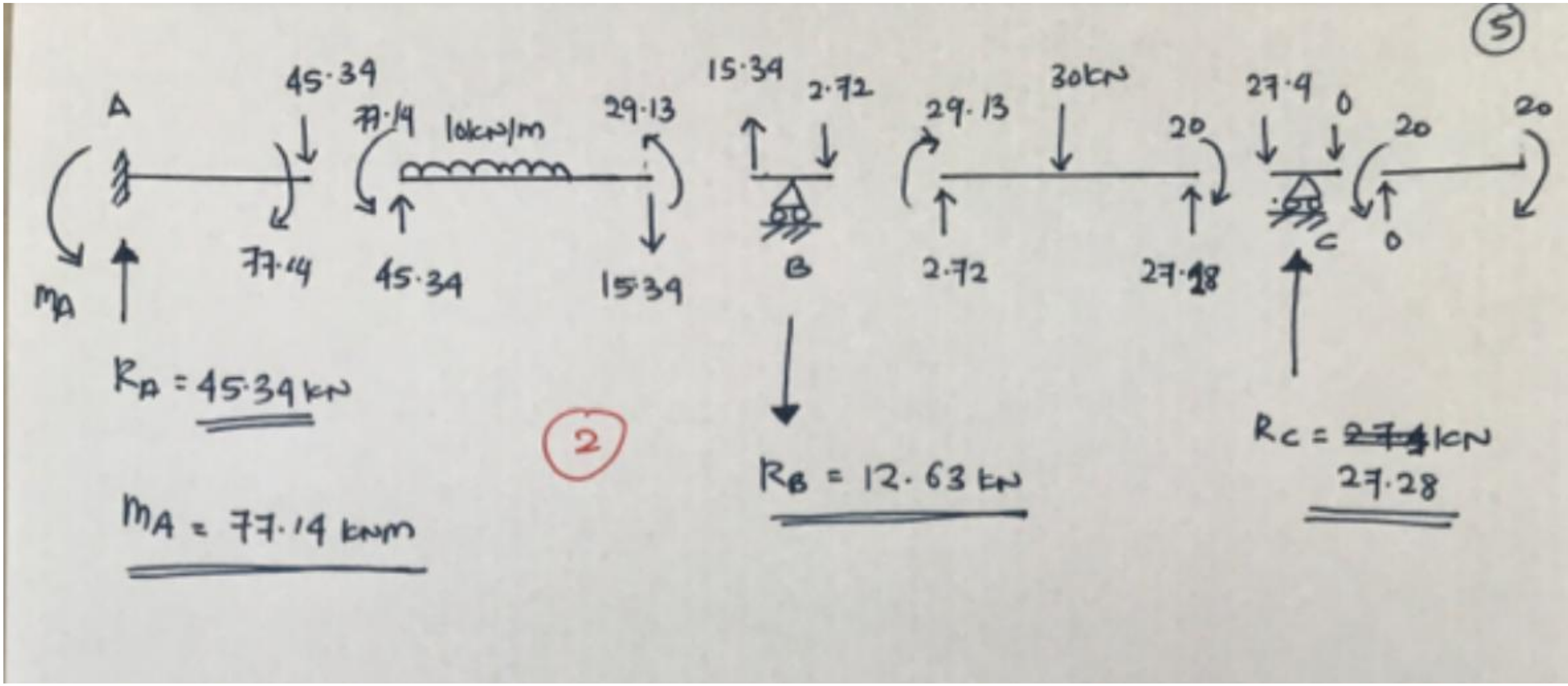
$\Sigma F_y = 0,$

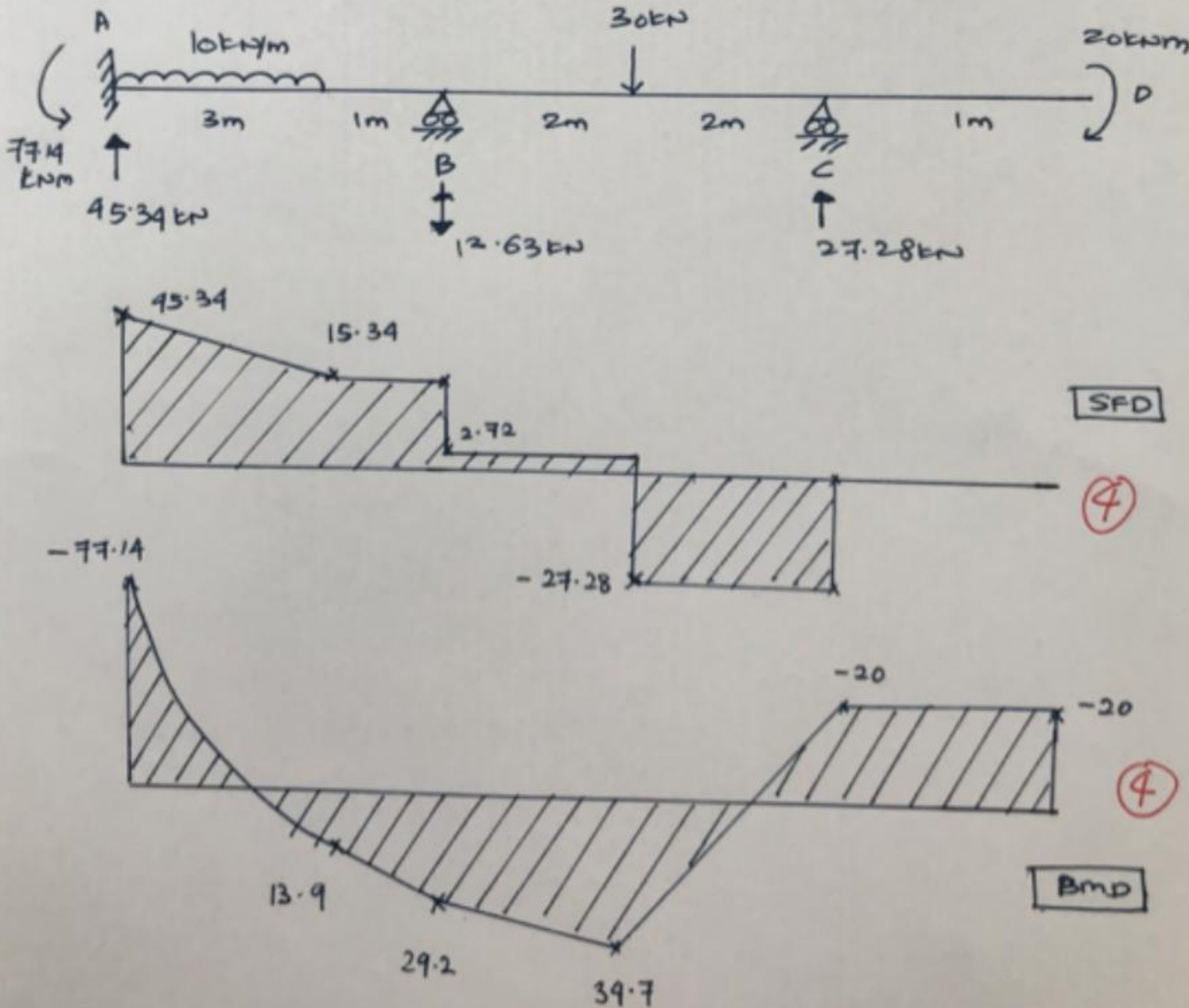
$2.72 - 30 + V_{CB} = 0,$

$V_{CB} = 27.28 \text{ kN}$

member CD

$V_{CD} = 0,$


5
A
45·34
77·14
10kN/m
29·13
15·34
2·72
29·13
30kN
20
27·4
0
20
20
mA
77·14
45·34
15·34
B
2·72
27·28
C
0
RA = 45·34 kN
2
RB = 12·63 kN
Rc = ~~27·4~~ kN
27·28
mA = 77·14 kNm


A
10kN/m
30kN
20kNm
D
3m
1m
2m
2m
1m
77.14 kNm
45.34kN
B
12.63kN
C
27.28kN
45.34
15.34
2.72
SFD
-27.28
4
-77.14
-20
-20
4
13.9
29.2
39.7
BMD